

Course: Math 458-01 (8387) - Theory of Numbers (3 credits)

Description: This course is an introduction to elementary theory of numbers. Topics include, but are not limited to: divisibility and prime numbers, Euclidean algorithm and applications, congruence arithmetic, primitive roots, quadratic residues, continued fractions, Diophantine linear and quadratic equations, approximations by rationals.

Pre-requisites: Aptitude for mathematics and mathematical curiosity. Some familiarity with formal proofs will be helpful; these skills will be developed during the course. Formal pre-requisites: [MATH 291 or CS 110] and [MATH 260 or MATH 314]. Please contact me if you don't have the formal pre-requisites but are interested in taking the course.

Audience: Math majors, both those planning on graduate school and those planning to teach high school. Computer Science majors looking for a theoretical elective. High school math teachers. Anyone who likes mathematics and wants to learn more.

Instructor: Catalin Zara, Professor of Mathematics.
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Website: czara.aczsite.net

Goals: By fully participating in all course activities, students should be able to:

- Understand the fundamental concepts of number theory;
- Enjoy learning number theory;
- Practice reading and writing mathematics.
- Appreciate the beauty and power of mathematics.

Expectations: Students enrolled in this course are expected to be:

- Motivated and disciplined;
- Adequately familiar with background material;
- Committed and actively involved in their own learning;
- Able to work in groups;
- Understanding the benefit of asking for help when needed.

Schedule: TuTh 4:00pm - 5:15pm in Y04-4110.
For every hour in class, you should dedicate at least two additional hours studying for this course.

Textbook: Lecture Notes - provided by the instructor, posted on *Blackboard*.
<https://umb.umassonline.net/>

Office hours: TuTh 1:00pm - 1:45pm in W03-154-15,
 TuTh 3:20pm - 3:50pm in Y04-4110 and 5:30pm - 6:15pm in TBD.
 We will be using *Blackboard* for class discussion. Rather than emailing questions to me, I strongly encourage you to post your questions on *Blackboard*.

Assignments: *Exams:* There will be two exams on (tentatively) March 23rd and May 4th. Both exams will have an in-class and a take-home component. Make-up exams will be allowed only with an official excuse. In all other situations, a missed exam will get a score of zero.

Homework: There will be about seven homework assignments, due roughly every other week. Late homework will be penalized.

Grading and cut-off scores:	First exam:	25 points	A : 90%
	Second exam:	25 points	B : 80%
	Homework:	45 points	C : 70%
	Participation:	5 points	D : 60%

Attendance: Regular class attendance is required and active class participation is expected. Students are responsible for material and announcements missed due to an absence. Please come to class on time and turn off your cell phone before the class begins.

Academic Integrity and Student Code of Conduct: Education at UMass Boston is sustained by academic integrity. Academic integrity requires that all members of the campus community are honest, trustworthy, responsible, respectful, and fair in academic work at the university. As part of being educated here, students learn, exercise, increase, and uphold academic integrity. Academic integrity is essential within all classrooms, in the many spaces where academic work is carried out by all members of the UMass Boston community, and in our local and global communities where the value of this education fulfills its role as a public good. Students are expected to adhere to the Student Code of Conduct, including policies about academic integrity, delineated in the University of Massachusetts Boston Graduate Studies Bulletin, Undergraduate Catalog, and relevant program student handbook(s), linked at www.umb.edu/academics/academic_integrity.

Special accommodations:

UMass Boston is committed to creating learning environments that are inclusive and accessible. If you have a personal circumstance that will impact your learning and performance in this class, please let me know as soon as possible, so we can discuss the best ways to meet your needs and the requirements of the course. If you have a documented disability, or would like guidance about navigating support services, contact the Ross Center for Disability Services by email (ross.center@umb.edu), phone (617-287-7430), or in person (Campus Center, UL Room 211). To receive accommodations, students must be registered with the Ross Center and must request accommodations each semester that they are in attendance at UMass Boston. For more information visit: www.rosscenter.umb.edu. Please note that the Ross Center will provide a letter for your instructor with information about your accommodation only and not about your specific disability.

Additional Resources:

UMass Boston is a vibrant, multi-cultural, and inclusive institution committed to ensuring that all members of our diverse campus community are able to thrive and succeed. The university provides a wide variety of resources to support students overall success. As we continue to deal with the evolving impacts of the COVID-19 pandemic, these resources are more important than ever.

- Are you in emotional distress? Call 617.287.5690 to speak with a licensed clinician 24/7 who can offer support, crisis recommendations, and assistance with finding resources.
- Have a campus question or issue? Use Here4U in the UMass Boston app or via www.umb.edu/here4U.
- Want advice in navigating a university or life situation? Contact the Dean of Students Office at www.umb.edu/deanofstudents.
- Want to connect with housing and food insecurity support, student life groups and events, or recreation activities? Visit www.umb.edu/life.
- Want to access resources specifically for immigrant-origin, DACA, TPS, and undocumented students? Visit www.umb.edu/immigrant.
- Looking for additional identity-based community support? Find more resources at www.umb.edu/identity-support
- Want to make the most of your academic experience? Visit www.umb.edu/academics/vpass/academic_support.
- Unable to attend class on a specific date or participate in an exam or class requirement due to a religious observance? Fill out the excused absence form (requires 2-weeks notice) to request religious accommodation at www.umb.edu/religiousabsence.

Tentative schedule

Date	Topics	Comments
Jan 24	Introduction. Numbers. Divisibility.	
Jan 26	GCD. Euclidean Algorithm.	Add/drop: Jan 30
Jan 31	Rational Numbers.	
Feb 2	Irreducible and Prime Numbers.	
Feb 7	The Fundamental Theorem of Arithmetics.	HW #1 due
Feb 9	A Bit of Analytic Number Theory.	
Feb 14	Linear Diophantine Equations.	
Feb 16	Linear Congruences.	
Feb 21	Periodicity of Powers.	HW #2 due
Feb 23	Euler's Totient Function.	
Feb 28	Primality Testing.	
Mar 2	Primitive Roots.	
Mar 7	Quadratic Residues.	HW #3 due
Mar 9	Quadratic Reciprocity.	
Mar 13	. Spring Break.	
Mar 15	. Spring Break.	
Mar 21	General Quadratic Residues.	
Mar 23	. Exam #1.	
Mar 28	Sums of Squares.	HW #4 due
Mar 30	More Sums of Squares.	
Apr 4	Brahmagupta-Pell Equations.	
Apr 6	Finite Continued Fractions.	
Apr 11	Periodic Continued Fractions.	HW #5 due
Apr 13	Fundamental Solutions.	
Apr 18	Diophantine Approximations.	
Apr 20	Binomial Coefficients.	P/W/F: Apr 20
Apr 25	Valuations.	HW #6 due
Apr 27	Gaussian Integers.	
May 2	Gaussian Primes.	
May 4	. Exam #2.	
May 9	Review and Further Directions.	HW #7 due